Solving Equations

Often we have a formula where we know values for some, but not all, of the variables. For example

Distance = Rate \times \text{Time}

Distance = 340 \text{ kilometers, Time} = 3 \text{ hours}

or

\[\frac{3.4}{\text{Rate}} = 27 \text{ D}\]

\[D = \frac{45 \text{ sec}}{\text{km}}\]

or

\[3X - 5Y = 6\]
\[2X + 8Y = 9\]

In each of these cases there is one equation for every unknown quantity. The goal is to rearrange the values so that an unknown value appears on one side, alone, and only known values appear on the other. The way to do this without destroying the information embodied in the equation is to use equal treatment for both sides of the equation.

You can do the following without making an equation untrue:

- Substitute a numerical value for a variable, if you know the value
- Multiply or divide by something, both sides (using zero here is legal, but it destroys the process)
- Add or subtract something, both sides
- Raise something to a power (like square it, take the square root)
- Clear parentheses, for example \(a(b+c) = ab + bc\)

and you should do one or another of the above (or several) if it helps to get all the known things on one side, and the unknown things on the other.

A few words about algebra. It is not very intuitive. Don’t expect to know the answer automatically, no matter how brilliant you are. It is rather like fixing a car or cooking. No matter how smart you are, it still takes all the steps and a bunch of time. It doesn’t mean that you are dumb or bad at math. It just works slowly and systematically.

You should also be aware that not everything you may do will automatically help isolate the unknown. If it doesn’t help, it may be necessary to start again. Even if it seems to have helped, it may be worthwhile to put the problem away. Forget what you have done. Then do it again to see whether it comes out the same (or whether you have made an error). It is really hard to find an error while you remember what you did before.

If you find algebra difficult, write out EVERY step. Then it will be easier to see whether both sides are treated equally.

Solving the second example

\[\frac{3.4}{\text{Rate}} = 27 \text{ D}\]
\[D = 45\]

To get the variable “Rate” off the bottom of the left hand side, multiply both sides of the equation by Rate i.e.

\[\frac{3.4}{\text{Rate}} = 27 \text{ D}\]
\[\text{Rate} \times \frac{3.4}{\text{Rate}} = 27 \text{ D Rate}\]
Now multiply both sides of the equation by \((1/27D)\) to get Rate by itself
\[
(1/27 \ D) \times 3.4 = 27 \ D \ Rate \ (1/27D)
\]
\[
(1/27 \ D) \times 3.4 = Rate
\]
And now substitute using \(D=45\text{sec/km}\) to get
\[
3.4/(27 \times 45\text{km/sec}) = Rate
\]
\[
0.002984 \text{ km/sec} = Rate = 2.984 \times 10^{-3} \text{ km/sec}
\]
For example, solving the two equations
\[
3X-5Y = 6
\]
\[
2X+8Y=9
\]
To isolate the variables, first get one of the equations to have only one of the variables. Suppose that you want an equation with only \(Y\). If you could get each of the equations to include \(3X\), then they could be subtracted and the values of \(X\) could be got rid of.
So multiplying both sides of
\[
2X+8Y=9
\]
by \((3/2)\) produces
\[
(3/2)2X+(3/2)8Y = (3/2)9
\]
(every term is multiplied by the same factor)
Working out the fractions
\[
3X+12Y = 27/2
\]
Now subtract from the other equation to produce
\[
3X - 5Y = 6
\]
\[
-(3X+12Y=27/2)
\]
which produces
\[
3X-3X -5Y-12Y = 6-27/2
\]
\[
-17Y = -15/2
\]
Now dividing by \(-17\) on both sides will leave \(Y\) by itself
\[
Y = 15/34 \ (\text{whew!})
\]
Now that \(Y\) is known, it can be substituted into either equation to leave \(X\) by itself.
\[
3X - 5Y = 6
\]
\[
Y = 15/34
\]
\[
3X - 5(15/34) = 6
\]
\[
3X = 6 + 5(15/34)
\]
\[
3X = 204/34 + 75/34
\]
\[
3X = 279/34
\]
\[
X = 93/34 \ (\text{finally})
\]